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Temporal Solitons in Nonlinear Media Modeled by Modified Complex Ginzburg Landau Equation Under Collective Variable Approach

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Abstract In the present paper, we have investigated the possibility of the existence of soliton solution in media modeled by the modified complex Ginzburg Landau equation. We have employed the technique of collective variables (CVs) to obtain a set of six coupled ordinary differential equations, one each for all the CVs included in the ansatz for the pulse. The coupled differential equations for the collective variables have been numerically solved to reveal the pulse dynamics which show stable soliton propagation.

Keywords Optical solitons · Modified complex Ginzberg Landau equation · Collective variables

1 Introduction

The evolution of a host of ideal nonlinear physical systems like optical pulses in nonlinear fiber optics communication systems, waves on water surface, plasma waves, Bose Einstein Condensate etc. are governed by the nonlinear Schrödinger equation (NLSE) [1–4]. NLSE is completely integrable admitting one or more soliton solution. For actual physical systems the NLSE model is only an approximation, additional terms need to be considered in the evolution equation to account for the various perturbations at work and corresponding compensations. One thing that is common among various nonlinear physical systems is that they give rise to energy localization effects with long life in space or time or in both, leading to the concept of spatial, temporal or spatiotemporal solitons. Solitons have been observed in various physical domains varying from long solitary waves of a few meters on a water surface to mili or micrometer optical pulses from a laser. Optical solitons have been the subject of extensive theoretical and experimental investigations because of their potential for various applications e.g. in long and short distance information communication systems, ultra fast switching systems etc. [5–13]. Solitons result from a balance between the linear and nonlinear response of the medium. The linear response of the medium results in the

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disintegration of a propagating pulse due to the group velocity dispersion and/or spatial diffraction. The nonlinearity of the medium compensates the linear dispersive and diffractive effects with the result that the pulse propagates without distortion.

In long distance communication systems various dissipative effects are inherently at work resulting in loss in pulse field. In order to take both the dispersive and dissipative effects into account, the physical media need to be modeled by a non-Hamiltonian system e.g. by the one dimensional complex Ginzburg Landau Equation (1D CGLE). For nonequilibrium systems exhibiting dispersive and dissipative effects, the 1D CGLE needs to be modified to include small nonlinear gradient terms resulting in 1D MCGLE [14–21]. For example, 1D MCGLE is a good model to observe the vacuum dissipative effects of the collective motion on top of a superfluid covariant nondissipative chaotic background. 1D MCGLE represents non Hamiltonian dissipative systems. These systems are much more complicated than the Hamiltonian ones because in addition to dispersion and nonlinearity terms, these systems contain terms which allow energy exchange with external sources. Though the non Hamiltonian systems or the dissipative systems are non integrable, nevertheless, soliton solutions of such systems do exist. One needs to employ any of the various approximation techniques in vogue. These include the inverse scattering transformation, the Hirota bilinear method, the Darboux-Backlund transform, Painleve expansion, Lagrangian variational method, variational iteration method, collective variable approach etc [8, 11, 12, 22–30].

The complex field $\Psi(z,t)$ of the 1D MCGLE follows the following equation

$$i\frac{\partial\Psi(z,t)}{\partial z}+p\frac{\partial^2\Psi}{\partial t^2}+q|\Psi|^2\Psi=c\frac{\frac{\partial\Psi}{\partial t}\cdot\frac{\partial\Psi^*}{\partial t}}{\Psi^*}+d\nabla^2[(\Psi.\Psi^*)^{1/2}]\left(\frac{\Psi}{\Psi^*}\right)^{1/2}+i\gamma\Psi, \qquad (1)$$

∇^2 is the one dimensional Laplacian operator; p,q,c,d,γ are the system parameters which may be real or complex. Different combinations of these system parameters lead to various types of waves through the medium in which many contributing factors are simultaneously at work. Mohamadau et al. [31] have made an investigation on the possibility of soliton solution of the 1D MCGLE with complex p, q, c, d and real γ and have been successful in obtaining several special soliton solutions. They obtain explicit expressions for fixed amplitude, arbitrary amplitude and chirp free solutions. In a different study, Mohamadau et al. [32] have studied the modulation instability in the 1D MCGLE with the complex nature of all the system parameters p,q,c,d,γ and obtained the contribution of the modified terms in the well known Lange and Newell's criteria. Recently, using the method of paraxial ray approximation, Hong [33] has reported the existence of a family of stationary solitons in a system modeled by real values of p, q, c, d and purely imaginary γ. These solitons have been found to be robust against small perturbations in positive dispersion, positive nonlinearity and negative dispersion and negative nonlinearity regimes.

In this paper, employing the principles of collective variable theory, we present the soliton dynamics of 1D MCGLE in media modeled by complex system parameters p, q, c, d and gamma in terms of only a few degrees of freedom. In long distance communication systems, intense light field exhibit complicated dynamical processes, difficult to comprehend from a direct analysis of the electromagnetic field. In order to understand such complicated dynamical behaviour involving infinite degrees of freedom, models of simple mechanical systems are invoked. Suitable models which need only a few degrees of freedom to describe their dynamics are devised. If a correspondence between the original field and the mechanical system can be made, we are successful in reducing the original system having infinite number of degrees of freedom to an equivalent simpler system having only a few degrees of freedom. Then each degree of freedom of the mechanical system is associated with one variable of the pulse e.g., with amplitude, width, phase etc. All the variables representing one parameter of the pulse are collectively called 'Collective Variables' (CV). In the collective variable approach, the study of the dynamics of the complex field of the 1D MCGLE is reduced to the study of the dynamics of various pulse parameters. Thus, instead of solving a second order partial differential equation, we are required to solve coupled first order ordinary differential equations for each of the CVs.

2 The Collective Variable Method

Collective variable approach proposed by Boesch et al. [34] has been successfully applied to condensed matter systems e.g. to the nonlinear Klein Gordon systems. The method is modified for application onto pulse dynamics in nonlinear fiber optics modeled by nonlinear equations like NLSE, CGLE or its modified version. The CV approach for nonlinear partial differential equation in nonlinear fiber optics is much simpler than in condensed matter systems. This is because optical solitons are assumed to be classical systems resulting from an exact balance between the fiber group velocity dispersion and its intensity dependent refractive index. The total field $\Psi(z,t)$ of the pulse through the optical fiber modeled by (1) at space-time point z, t is assumed to be composed of the soliton field $f(z,t)$ through the fiber and residual field $g(z,t)$ which is responsible for the dressing of the soliton and any residual leakage coupled to the soliton motion.

$$\Psi(z,t) = f(z,t) + g(z,t). \tag{2}$$

A guess for the ansatz for the soliton field $f(z,t)$, which may be the best representation of the configuration of the pulse, is made. The field $f(z,t)$ can have Gaussian profile, hyperbolic secant, raised cosine profile or can be expansion in terms of Hermite Gaussian polynomials. The soliton field $f(z,t)$ is assumed to depend on a large number of variables of the pulse, like amplitude, frequency, width etc., together called collective variables $X_1, X_2, \dots, X_N$. With the introduction of CVs, the soliton field at any space-time position (z,t), becomes implicit function of space.

$$f(z,t) = f\{X_1(z), X_2(z), \dots, X_N(z), t\}. \tag{3}$$

Inclusion of CV into the ansatz for soliton field introduces extra degrees of freedom resulting in the expansion of the available phase space of the system. This would introduce undesirable solutions into the system. In order that the system remains in the same space as the original field equation, the system of new variables need to be constrained. The common constraint imposed in the CV approach is that the ansatz function for the soliton field is configured in such a way that the residual free energy (RFE) associated with the pulse envelope is minimum. RFE is defined as

$$\varepsilon = \int_{-\infty}^{+\infty} |g|^2 dt = \int_{-\infty}^{+\infty} |\Psi(z,t) - f\{X_1(z), X_2(z), \dots, X_N(z)\}|^2 dt. \tag{4}$$

The RFE serves as a measure for the correctness of the ansatz function for the soliton field. From the RFE, we construct two quantities C_j and $\dot{C}_j$, where C_j describes the rate of change of RFE w.r.t. jth CV and $\dot{C}_j$ describes the rate of change of C_j with normalized distance.

$$C_j = \frac{\partial \varepsilon}{\partial X_j} = \int_{-\infty}^{+\infty} g \frac{\partial g^*}{\partial X_j} dt + \text{c.c.}, \tag{5}$$

$$\dot{C}_j = \frac{d}{dz}\left(\int_{-\infty}^{+\infty} g f_{X_j}^* dt\right) + \text{c.c.}, \tag{6}$$

where c.c. stands for complex conjugate.

Using $g(z,t) = \Psi(z,t) - f\{X_1(z), X_2(z), \ldots, X_N(z), t\}$, the expression for $C_j$and $\dot{C}_j$ can be rewritten as

$$C_j = -\int_{-\infty}^{+\infty} g f_{X_j}^* dt + \text{c.c.}, \tag{7}$$

$$\dot{C}_j = -\int_{-\infty}^{+\infty} f_{X_j}^* g_z dt - \sum_{k=1}^{N} \int_{-\infty}^{+\infty} \frac{\partial}{\partial X_k}(f_{X_j}^*) \frac{\partial X_k}{\partial z} g dt + \text{c.c.}, \tag{8}$$

where the overhead dot implies differentiation w.r.t. z and the subscript X_j on f denotes differentiation w.r.t. X_j.

For optical solitons in fibers, the constraint implies that the system evolves only in a particular direction such that the RFE during the dynamics is weakly equal to zero. Following Dirac's theory of constrained dynamical system, a quantity which is weakly equal to zero cannot be set to zero until all the variations of the quantity w.r.t. the dynamical variations have been performed. All these variations result in the equations of motion for the CVs. Appropriate initial values of CVs are chosen so as to satisfy the constraint of minimizing RFE. This condition is obtained by setting

$$C_j \approx 0. \tag{9}$$

However, the constraints which are satisfied at the beginning at $z = 0$ may not be satisfied throughout the propagation distance z. Thus, a second set of constraint defined as

$$\dot{C}_j \approx 0, \tag{10}$$

ensures that the minimum value of RFE of the soliton is maintained throughout its propagation. The equations of motion for the collective variables which represent the evolution of different pulse parameters are obtained from the equations of constraints.

3 Evolution Equations for Collective Variables

For convenience, (1) is recasted in the following form,

$$\begin{aligned} &i\frac{\partial \Psi}{\partial z} + (p_r + ip_i)\frac{\partial^2 \Psi}{\partial t^2} + (q_r + iq_i)|\Psi|^2\Psi \\ &\quad = (c_r + ic_i)\frac{\frac{\partial \Psi}{\partial t}\frac{\partial \Psi}{\partial t}^*}{\Psi^*} + (d_r + id_i)\frac{[\frac{1}{2}\Psi\Psi^*\frac{\partial^2(\Psi\Psi^*)}{\partial t^2} - \frac{1}{4}(\frac{\partial(\Psi\Psi^*)}{\partial t})^2]}{\Psi\Psi^{*2}} + i(\gamma_r + i\gamma_i)\Psi. \end{aligned} \tag{11}$$

For the pulse field $\Psi(z,t)$, the sum of the soliton field and the residual field, (2) is substituted into (11) to get the evolution equation for the residual field, which is

$$\begin{aligned} \frac{\partial g}{\partial z} &= ip\frac{\partial^2 g}{\partial t^2} + iq|f+g|^2 g + \gamma g + ip\frac{\partial^2 f}{\partial t^2} + iq|f+g|^2 f + \gamma f - ic\frac{\frac{\partial(f+g)}{\partial t}\frac{\partial(f+g)}{\partial t}^*}{f+g} \\ &\quad - id\frac{\frac{1}{2}|f+g|^2\frac{\partial^2|f+g|^2}{\partial t^2} - \frac{1}{4}(\frac{\partial|f+g|^2}{\partial t})^2}{|f+g|^2(f+g)^*} - \sum_{i=1}^{N}\frac{\partial f}{\partial X_i}\dot{X}_i. \end{aligned} \tag{12}$$

Equation (12) when substituted into (8), gives

$$\dot{C}_j = \sum_{k=1}^{N}\left[2\Re\int_{-\infty}^{+\infty}\left(f^*_{X_j} f_{X_k}\right)dt - 2\Re\int_{-\infty}^{+\infty}\left(f^*_{X_j X_k} g\right)dt\right] - R_j, \quad \text{or,}$$
$$-\dot{C}_j = -\sum_{k=1}^{N}\frac{\partial C_j}{\partial X_k}\dot{X}_k + R_j, \quad \text{where} \tag{13}$$

$$\frac{\partial C_j}{\partial X_k} = 2\Re\int_{-\infty}^{+\infty} f^*_{X_j} f_{X_k} dt - 2\Re\int_{-\infty}^{+\infty} f^*_{X_j Xk} g dt. \tag{14}$$

$\Re$ in (13) stands for the real part, and R_j is given by (15).

$$\begin{aligned} R_j &= 2\Re\int_{-\infty}^{+\infty} ip\frac{\partial f^*}{\partial X_j}\frac{\partial^2 g}{\partial t^2}dt + 2\Re\int_{-\infty}^{+\infty} iq\frac{\partial f^*}{\partial X_j}|f+g|^2 g dt \\ &\quad - 2\Re\int_{-\infty}^{+\infty} ic\frac{\partial f^*}{\partial X_j}\frac{\frac{\partial(f+g)}{\partial t}\frac{\partial(f+g)^*}{\partial t}}{(f+g)^*}dt \\ &\quad + 2\Re\int_{-\infty}^{+\infty}\gamma\frac{\partial f^*}{\partial X_j} g dt + 2\Re\int_{-\infty}^{+\infty} ip\frac{\partial f^*}{\partial X_j} f_{tt} dt \\ &\quad + 2\Re\int_{-\infty}^{+\infty} iq\frac{\partial f^*}{\partial X_j}|f+g|^2 f dt + 2\Re\int_{-\infty}^{+\infty}\gamma\frac{\partial f^*}{\partial X_j} f dt \\ &\quad - 2\Re\int_{-\infty}^{+\infty} id\frac{\partial f^*}{\partial X_j}\frac{\frac{1}{2}\frac{\partial^2(|f+g|^2)}{\partial t^2}|f+g|^2 - \frac{1}{4}\{\frac{\partial|f+g|^2}{\partial t}\}^2}{|f+g|^2(f+g)^*}dt. \end{aligned} \tag{15}$$

Equation (13) is not just a single equation; they are as many as the number of collective variables considered in the ansatz for the soliton field. These N equations can collectively be written as a matrix equation.

$$-(\dot{C}) = -\left(\frac{\partial C}{\partial X}\right)(\dot{X}) + R, \quad \text{where}$$
$$(C) \equiv \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ \vdots \\ C_N \end{pmatrix}, \quad (X) \equiv \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ \vdots \\ X_N \end{pmatrix}, \quad (R) \equiv \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ \vdots \\ R_N \end{pmatrix},$$
$$\left(\frac{\partial C}{\partial X}\right) \equiv \begin{pmatrix} (\frac{\partial C_1}{\partial X_1}) & (\frac{\partial C_1}{\partial X_2}) & \cdots & (\frac{\partial C_1}{\partial X_N}) \\ (\frac{\partial C_2}{\partial X_1}) & (\frac{\partial C_2}{\partial X_2}) & \cdots & (\frac{\partial C_2}{\partial X_N}) \\ \vdots & \vdots & & \vdots \\ (\frac{\partial C_N}{\partial X_1}) & (\frac{\partial C_N}{\partial X_2}) & \cdots & (\frac{\partial C_N}{\partial X_N}) \end{pmatrix}. \tag{16}$$

By virtue of the equations of constraints $(\dot{C}) = 0$, equations of motion for the collective variables are obtained.

$$(\dot{X}) = \left(\frac{\partial C}{\partial X}\right)^{-1} (R).$$

Or equivalently,

$$\begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \vdots \\ \dot{X}_N \end{pmatrix} = \begin{pmatrix} \frac{\partial C_1}{\partial X_1} & \frac{\partial C_1}{\partial X_2} & \frac{\partial C_1}{\partial X_3} & \cdots & \frac{\partial C_1}{\partial X_N} \\ \frac{\partial C_2}{\partial X_1} & \frac{\partial C_2}{\partial X_2} & \frac{\partial C_2}{\partial X_3} & \cdots & \frac{\partial C_2}{\partial X_N} \\ \frac{\partial C_3}{\partial X_1} & \frac{\partial C_3}{\partial X_2} & \frac{\partial C_3}{\partial X_3} & \cdots & \frac{\partial C_3}{\partial X_N} \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial C_N}{\partial X_1} & \frac{\partial C_N}{\partial X_2} & \frac{\partial C_N}{\partial X_3} & \cdots & \frac{\partial C_N}{\partial X_N} \end{pmatrix}^{-1} \begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ \vdots \\ R_N \end{pmatrix}. \tag{17}$$

The soliton field through the fiber is assumed to have Gaussian form. Six collective variables $X_1(z), X_2(z), \ldots, X_6(z)$are introduced into the Gaussian ansatz

$$f(X_1(z), \ldots, X_6(z), t) = X_1 \exp\left[-\frac{(t - X_2)^2}{X_3^2} - i\frac{X_4}{2}(t - X_2)^2 + iX_5(t - X_2) + iX_6\right]. \tag{18}$$

The collective variables $X_1(z)$, $X_2(z)$, $X_3(z)$, $X_4(z)$, $X_5(z)$, $X_6(z)$ represent the amplitude, temporal position, width, chirp, frequency and phase of the pulse respectively. Equations for all the collective variables are obtained under the lowest order CV theory also known as 'bare approximation'. Under the bare approximation, dressing of the soliton and any radiation associated with its propagation is assumed to be negligible. This is incorporated by setting the residual field $g(z, t)$ equal to zero. Matrices $(\frac{\partial C}{\partial X})$ and (R) are obtained as

$$\left(\frac{\partial C}{\partial X}\right) = \sqrt{2\pi} \begin{pmatrix} X_3 & 0 & \frac{X_1}{2} & 0 & 0 & 0 \\ 0 & \frac{X_3^3}{4}(\frac{4X_1^2}{X_3^4} + X_1^2 X_4^2 + X_1^2 X_3 X_5^2) & 0 & -\frac{X_1^2 X_3^3 X_5}{8} & -\frac{X_1^2 X_3^3 X_4}{4} & -X_1^2 X_3 X_5 \\ \frac{X_1}{2} & 0 & \frac{3X_1^2}{4X_3} & 0 & 0 & 0 \\ 0 & -\frac{X_1^2 X_3^3 X_5}{8} & 0 & \frac{3X_1^2 X_3^5}{64} & 0 & \frac{X_1^2 X_3^3}{8} \\ 0 & -\frac{X_1^2 X_3^3 X_4}{4} & 0 & 0 & \frac{X_1^2 X_3^3}{4} & 0 \\ 0 & -X_1^2 X_3 X_5 & 0 & \frac{X_1^2 X_3^3}{8} & 0 & X_1^2 X_3 \end{pmatrix}, \tag{19}$$

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \end{pmatrix} = \sqrt{2\pi} \begin{pmatrix} [p_i(-\frac{X_1}{X_3} - \frac{X_1X_3^3X_4^2}{4} - X_1X_3X_5^2) + q_i\frac{X_1^3X_3}{\sqrt{2}} - ci(\frac{X_1}{X_3} + \frac{X_1X_3^3X_4^2}{4} + X_1X_3X_5^2) + d_i\frac{X_1}{X_3} - \gamma_r X_1X_3] \\ [-p_r(\frac{3X_1^2X_5}{X_3} + \frac{3X_1^2X_3^3X_4^2X_5}{4} + X_1^2X_3X_5^3) + q_r\frac{X_1^4X_3X_5}{\sqrt{2}} - c_r(\frac{3X_1^2X_3^3X_4^2X_5}{4} + \frac{X_1^2X_5}{X_3} + X_1^2X_3X_5^3) \\ -c_iX_1^2X_3X_4X_5 + d_r\frac{X_1^2X_5}{X_3} + \gamma_i X_1^2X_3X_5] \\ [-p_rX_1^2X_4 + p_i(\frac{X_1^2}{2X_3^2} - \frac{3X_1^2X_3^2X_4^2}{4} - \frac{X_1^2X_5^2}{2}) + q_i\frac{X_1^4}{4\sqrt{2}} - ci(\frac{3X_1^2}{2X_3^2} + \frac{3X_1^2X_3^2X_4^2}{8} + \frac{X_1^2X_5^2}{2}) \\ -d_i\frac{X_1^2}{2X_3^2} - \gamma_r\frac{X_1^2}{2}] \\ [p_r(-\frac{X_1^2X_3}{8} + \frac{3X_1^2X_3^5X_4^2}{32} + \frac{X_1^2X_3^3X_5^2}{8}) - p_i\frac{X_1^2X_3^3X_4}{4} - q_r\frac{X_1^4X_3^3}{16\sqrt{2}} + c_r(\frac{3X_1^2X_3}{8} + \frac{3X_1^2X_3^5X_4^2}{32X_3} \\ + \frac{X_1^2X_3^3X_5^2}{8}) + d_r\frac{X_1^2X_3}{8} - \gamma_i\frac{X_1^2X_3^3}{8}] \\ [p_r\frac{X_1^2X_3^3X_4X_5}{2} - p_iX_1^2X_3X_5 + c_r\frac{X_1^2X_3^3X_4X_5}{2}] \\ [p_r(\frac{X_1^2}{X_3} + \frac{X_1^2X_3^3X_4^2}{4} + X_1^2X_3X_5^2) - q_r\frac{X_1^4X_3}{\sqrt{2}} + c_r(\frac{X_1^2}{X_3} + \frac{X_1^2X_3^3X_4^2}{4} + X_1^2X_3X_5^2) - d_r\frac{X_1^2}{X_3} - \gamma_iX_1^2X_3] \end{pmatrix}. \tag{20}$$

By virtue of (18), (19) and (20), finally (17) leads to the following nonlinear dynamical system.

$$\dot{X}_1 = -p_rX_1X_4 + p_i\left(\frac{2X_1}{X_3^2} + X_1X_5^2\right) - q_i\frac{5X_1^3}{4\sqrt{2}} + c_iX_1X_5^2 - d_i\frac{2X_1}{X_3^2} + \gamma_rX_1, \tag{21}$$

$$\dot{X}_2 = p_iX_3^2X_4X_5 + p_r2X_5 + c_iX_3^2X_4X_5, \tag{22}$$

$$\dot{X}_3 = p_r2X_3X_4 + p_i\left(\frac{X_3^3X_4^2}{2} - \frac{2}{X_3}\right) + q_i\frac{X_1^2X_3}{2\sqrt{2}} + c_i\left(\frac{2}{X_3} + \frac{X_3^3X_4^2}{2}\right) + d_i\frac{2}{X_3}, \tag{23}$$

$$\dot{X}_4 = p_r2\left(\frac{4}{X_3^4} - X_4^2\right) + p_i\frac{8X_4}{X_3^2} - q_r\frac{\sqrt{2}X_1^2}{X_3^2} - 2c_r\left(\frac{4}{X_3^4} + X_4^2\right) - d_r\frac{8}{X_3^4}, \tag{24}$$

$$\dot{X}_5 = p_i\left(\frac{4X_5}{X_3^2} + X_3^2X_4^2X_5\right) - c_r2X_4X_5 + c_iX_3^2X_4^2X_5, \tag{25}$$

$$\dot{X}_6 = p_r\left(X_5^2 - \frac{2}{X_3^2}\right) - p_i\left(X_4 - X_3^2X_4X_5^2\right) + q_r\frac{5X_1^2}{4\sqrt{2}} - c_rX_5^2 + c_iX_3^2X_4X_5^2 + d_r\frac{2}{X_3^2} + \gamma_i. \tag{26}$$

4 Results and Conclusion

We have carried out numerical investigation of evolution equations which are coupled ordinary differential equations (ODE). These ODEs have been integrated using the standard fourth order Runge Kutta Method. Without any loss of generality, the initial pulse amplitude and the pulse width have been chosen to have value unity and the initial temporal position, chirp, frequency and phase have been taken to be equal to zero. For the chosen values of

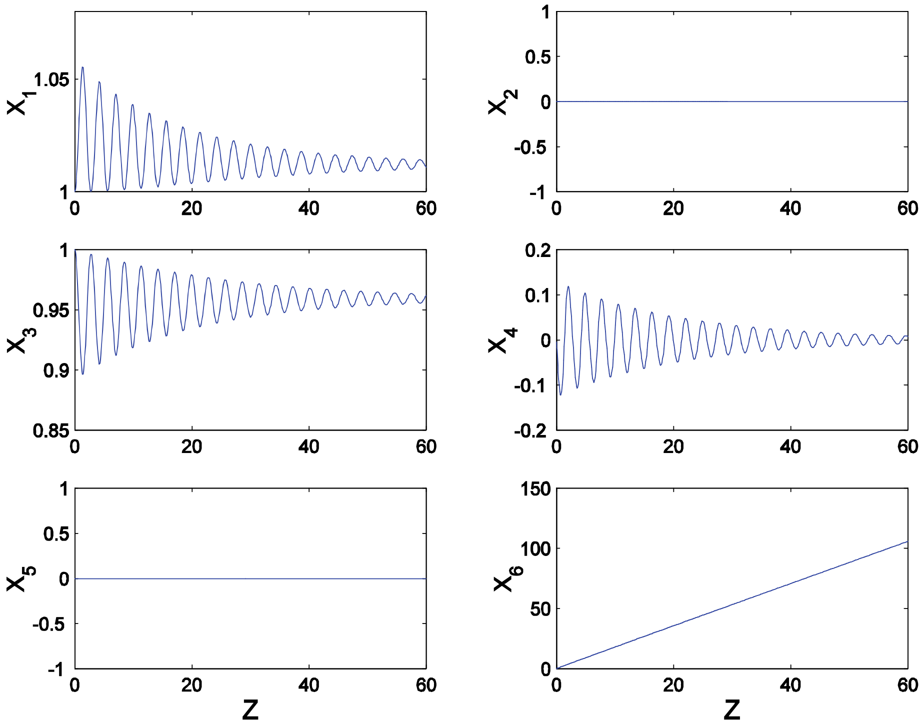

Fig. 1 Variations of the pulse parameters X_1, X_2, X_3, X_4, X_5 and X_6 with distance of propagation. $X_1(0) = 1.0$, $X_2(0) = 0$, $X_3(0) = 1$, $X_4(0) = 0$, $X_5(0) = 0$ and $X_6(0) = 0$. $p_r = 0.5$, $p_i = -0.02$, $q_r = 3.0$, $q_i = -0.001$, $c_r = -0.01$, $c_i = -0.01$, $d_r = 0.01$, $d_i = -0.01$, $\gamma_r = 0.021$, $\gamma_i = 0.1$

the various system parameters, amplitude (X_1), pulse width (X_3), chirp (X_4) are found to vary periodically as the pulse propagates. For one set of system parameters, the periodic vibration is damped sinusoidal. After the initial variation, the amplitude settles to a value a little higher than the initially chosen one while the width, showing a similar variation settles down to a little lower value as the pulse moves forward. Chirp in the pulse which changes by large amount at the beginning, becomes zero afterwards. The temporal position (X_2) and frequency (X_5) remain the same throughout the propagation while the phase (X_6) of the pulse shows linear increase as it moves forward. In higher dispersive media, variations of amplitude, width and chirp again are periodic, the changes being more complex, but this time also the variations settle down to constant values. Total energy of the pulse remains constant throughout the propagation. In the first case, after initial fluctuations settles down, the amplitude is found to increase over the initial value while there is a corresponding decrease in pulse width. The pulse adjusts itself in a manner that a redistribution of its field energy takes place with amplitude increasing, width decreasing such that its total energy is conserved. In the second case, in a medium showing higher dispersive effect, a redistribution of total field energy takes place resulting in a decrease in amplitude with a corresponding increase in pulse width. Chirp again dies down in the steady state propagation.

To conclude, we have been successful in solving the 1D MCGLE which includes among other terms, a nonlinear gradient term signifying non equilibrium system. We obtained the evolution equations for the CVs for the propagation of optical solitons through such media under bare approximation, ignoring the effect of residual field. The CV approach has a very

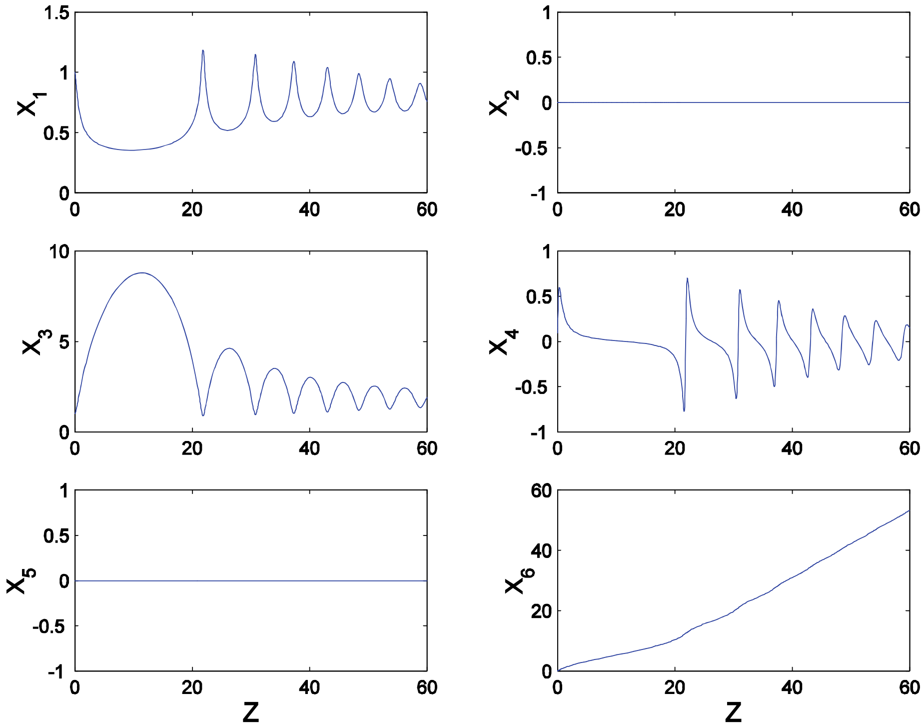

Fig. 2 Variations of the pulse parameters X_1, X_2, X_3, X_4, X_5 and X_6 with distance of propagation. $X_1(0) = 1.0$, $X_2(0) = 0$, $X_3(0) = 1$, $X_4(0) = 0$, $X_5(0) = 0$ and $X_6(0) = 0$. $p_r = 1.0$, $p_i = -0.045$, $q_r = 3.0$, $q_i = -0.005$, $c_r = -0.01$, $c_i = -0.01$, $d_r = 0.01$, $d_i = -0.01$, $\gamma_r = 0.01$, $\gamma_i = 0.1$

wide applicability, it can be employed both for conservative and non conservative systems. It is not difficult to derive the CV equations of motion for any system possessing nonlinearity of higher order and various dissipative terms. In this approach, contribution of every term in the field equation which governs the pulse dynamics can be observed separately.

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